Lenticular Poles of the Dynamical Zeta Funtion of the beta-shift for simple Parry numbers close to one and Lehmer's problem

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aim of the work : A proof of the Conjecture of Lehmer and of the Conjecture of Schinzel-Zassenhaus by the β-shift, for $\beta > 1$ tending to one among **real** reciprocal algebraic integers > 1.

sufficient for all reciprocal nonzero (complex) algebraic integers which are not roots of unity : for α be an algebraic integer, the house

$\beta := \overline{|\alpha|}$ is a **real** algebraic integer.

Assume $\beta > 1$. Consider the β-shift.

Lehmer '33 : does there exist a minorant > 1 of M(α) ? for all nonzero algebraic integer α and not a root of unity.

does there exist a minorant > 1 of M(β) ? for all reciprocal algebraic integer $\beta > 1$ when β tends to 1.

oOo

Schinzel-Zassenhaus '65 :
There exists a constant $C > 0$ such that, for α any nonzero algebraic integer which is not a root of unity, then

$$\beta := \lceil\overline{\alpha}\rceil \geq 1 + \frac{C}{\deg(\alpha)}.$$

The β-shift allows to obtain a **continuous minorant** > 1 **of M**(β), by putting into evidence the origin of the problem of Lehmer, i.e. the **existence of lenticuli of conjugates of** β**.**

The β-shift allows to obtain **structure theorems** on the minimal polynomials P_β having Mahler measure $\mathrm{M}(\beta)$ less than the number of Lehmer $1.1762\ldots$.

The β-shift allows to obtain a new (more natural) Dobrowolsky type inequality with the dynamical degree $\mathrm{dyg}(\beta)$ which replaces the usual degree $\deg(\beta)$.

The β-shift is coupled with the introduction of a new method of **divergent series** (asymptotic expansions à la Poincaré (1895) in Celestial Mechanics) to obtain asymptotic expressions of the roots of the trinomials

$$G_n(x) = -1 + x + x^n, \qquad n \geq 6.$$

Why these trinomials are so important ?

let θ_n be the unique root of the trinomial $G_n(z) := -1 + z + z^n$ in $(0, 1)$.

θ_n^{-1} $\quad$ $-1 + z + z^n$

$\theta_n^{-1} < \beta < \theta_{n-1}^{-1}$ $\quad$ $-1 + x + x^n + x^{m_1} + x^{m_2} + \ldots + x^{m_s} + \ldots$

where $m_1 - n \geq n - 1$, $m_{q+1} - m_q \geq n - 1$

θ_{n-1}^{-1} $\quad$ $-1 + z + z^{n-1}$

Reciprocal algebraic integers β are never simple Parry numbers.

(θ_n^{-1}) tends to 1. β tends to 1 equivalently n tends to infinity.

$$\theta_n^{-1} < \beta < \theta_{n-1}^{-1} \qquad -1 + x + x^n + x^{m_1} + x^{m_2} + \ldots + x^{m_s} + \ldots$$

where $s \geq 1$, $m_1 - n \geq n-1$, $m_{q+1} - m_q \geq n-1$ for $1 \leq q < s$

denoted $=: f_\beta(x)$, is the inverse of the dynamical zeta function $\zeta_\beta(z)$ (up to the sign) of the β-shift, equivalently is the generalized Fredholm determinant of the Perron-Frobenius operator associated with the β-transformation. Called **Parry Upper function** at β.

its zeroes := eigenvalues of the transfer operators, := poles of $\zeta_\beta(z)$.

Method : 1) M(θ_n^{-1}) from the lenticuli of roots in $|\arg(z)| < \pi/3$, with n tending to ∞,
2) extended to any $\theta_n^{-1} < \beta < \theta_{n-1}^{-1}$, with reduced lenticuli of roots, and n tending to ∞.

n is by definition the dynamical degree of β, denoted by $\mathrm{dyg}(\beta)$.

$$\mathrm{M}(\beta) := \prod_i \max\{1, |\beta^{(i)}|\}$$

$\mathrm{M}(\beta) = \mathrm{M}(\beta^{-1})$; then consider all the zeroes of the minimal polynomials P_β inside the unit disk. How to pass

from the zeroes of $f_\beta(x)$ to the zeroes of $P_\beta(x)$ of modulus < 1 ?

only a certain proportion of zeroes can be identified as conjugates of β : the **lenticular zeroes.**

minorant :

$$\mathrm{M}_r(\beta) =: \mathrm{M}_{lenticulus}(\beta) := \prod_{i \ lenticular} \min\{1, |\beta^{(i)}|^{-1}\}$$

expressed as an symptotic expansion of $\mathrm{dyg}(\beta)$.

Let $\kappa := 0.171573\ldots$ be the value of the maximum of the function $a \to \kappa(1,a) := \frac{1-\exp(\frac{-\pi}{a})}{2\exp(\frac{\pi}{a})-1}$ on $(0,\infty)$. Let $S := 2\arcsin(\kappa/2) = 0.171784\ldots$. Denote

$$\Lambda_r\mu_r := \exp\Big(\frac{-1}{\pi}\int_0^S \mathrm{Log}\Big[\frac{1+2\sin(\frac{x}{2}) - \sqrt{1-12\sin(\frac{x}{2})+4(\sin(\frac{x}{2}))^2}}{4}\Big]dx\Big)$$

$= 1.15411\ldots$, a value slightly below Lehmer's number $1.17628\ldots$

Theorem

$$\lim_{\mathrm{dyg}(\beta)\to\infty} \prod_{\omega\in\mathscr{L}_\beta} |\omega|^{-1} = \Lambda_r\mu_r.$$

It is the limit lenticular contribution of the Parry Upper function.

Further : **identify** the lenticuli of zeroes of the Parry Upper function at β with lenticuli of conjugates of β, so that

$$\lim_{\mathrm{dyg}(\beta)\to\infty} \prod_{\omega\in\mathscr{L}_\beta} |\omega|^{-1} = \lim_{\mathrm{dyg}(\beta)\to\infty} \mathrm{M}_r(\beta) = \Lambda_r \mu_r .$$

Further : the asymptotic expansions of the roots of $f_\beta(z)$ lying in β gives an asymptotic expansion of the lenticular minorant of the Mahler measure :

Theorem (Dobrowolski type minoration)

Let β be a nonzero algebraic integer which is not a root of unity such that $\mathrm{dyg}(\beta) \geq 260$. Then

$$\mathrm{M}(\alpha) \geq \Lambda_r \mu_r - \Lambda_r \mu_r \frac{S}{2\pi} \left(\frac{1}{\mathrm{Log}\,(\mathrm{dyg}(\beta))} \right)$$

JLVG, *On the Conjecture of Lehmer, Limit Mahler Measure of Trinomials and Asymptotic Expansions*, Uniform Distribution Theory J. **11** (2016), 79–139.

JLVG (Sept. 2017) : *A Proof of the Conjecture of Lehmer and of the Conjecture of Schinzel-Zassenhaus*

arXiv.org $>$ math $>$ arXiv :1709.03771

version v2 (2018).

D. Dutykh and **JLVG**, *On the Reducibility and the Lenticular Sets of Zeroes of Almost Newman Polynomials Having Lacunarity Controlled a Minima*, preprint (2018).

Lehmer's Conjecture :

Theorem (VG '17)

For any nonzero algebraic integer α which is not a root of unity,

$$\mathrm{M}(\alpha) \geq \theta_{259}^{-1} = 1.016126\ldots$$

Schinzel Zassenhaus's Conjecture :

Theorem (VG '17)

Schinzel-Zassenhaus's conjecture is true. Let α be a nonzero algebraic integer which is not a root of unity. Then

$$\lceil\overline{\alpha}\rceil \geq 1 + \frac{C}{\deg(\alpha)} \qquad \textit{with } C = \theta_{259}^{-1} - 1 = 0.016126\ldots.$$

Pf. : Let $\alpha \neq 0$ be an algebraic integer which is not a root of unity. Since $\mathrm{M}(\alpha) = \mathrm{M}(\alpha^{-1})$ there are three cases to be considered :
(i) the house of α satisfies $\overline{|\alpha|} \geq \theta_5^{-1}$,
(ii) the dynamical degree of α satisfies : $6 \leq \mathrm{dyg}(\alpha) < 260$,
(iii) the dynamical degree of α satisfies : $\mathrm{dyg}(\alpha) \geq 260$.

In case (i), $\mathrm{M}(\alpha) \geq \theta_5^{-1} \geq \theta_{259}^{-1}$ and the claim holds true.
In the second case, since $\mathrm{M}(\alpha)$ is the product of $\overline{|\alpha|}$ by the moduli of the conjugates of modulus > 1, we have $\mathrm{M}(\alpha) \geq \overline{|\alpha|}$, therefore $\mathrm{M}(\alpha) \geq \theta_{259}^{-1}$.
In case (iii), the Dobrowolski type inequality gives the following lower bound of the Mahler measure

$$\mathrm{M}(\alpha) \geq \Lambda_r \mu_r - \frac{\Lambda_r \mu_r \arcsin(\kappa/2)}{\pi \,\mathrm{Log}\,(\mathrm{dyg}(\alpha))} \geq \Lambda_r \mu_r - \frac{\Lambda_r \mu_r \arcsin(\kappa/2)}{\pi \,\mathrm{Log}\,(259)}, = 1.14843\ldots$$

This lower bound is numerically greater than $\theta_{259}^{-1} = 1.016126\ldots$.
Therefore, in any case, the lower bound θ_{259}^{-1} of $\mathrm{M}(\alpha)$ holds true. We deduce the general minorant on M.

1. A certain class of lacunary polynomials and their lenticular zero locus

2. Minimal polynomials versus Parry Upper functions in Lehmer's problem

Contents

Class $\mathscr{B}$ of lacunary polynomials

For $n \geq 2$, we study the factorization of the polynomials

$$f(x) := -1 + x + x^n + x^{m_1} + x^{m_2} + \ldots + x^{m_s}$$

where $s \geq 0$, $m_1 - n \geq n - 1$, $m_{q+1} - m_q \geq n - 1$ for $1 \leq q < s$. Denote by $\mathscr{B}$ the class of such polynomials, and by $\mathscr{B}_n$ those whose third monomial is exactly x^n, so that

$$\mathscr{B} = \cup_{n \geq 2} \mathscr{B}_n.$$

Objectives :

- factorization ?
- irreducibility ($m_s \to \infty$) ?
- Zero locus ?

lenticulus $\mathscr{L}_{\theta_{28}^{-1}}$ of simple zeroes in $\arg(z) \in (-\pi/3, +\pi/3)$

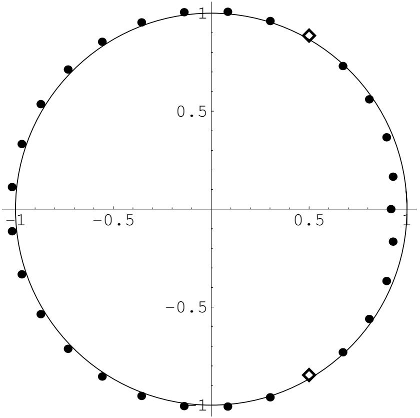

FIGURE: Roots of $G_{28}(z)$, $n = 28$.

lenticulus $\mathscr{L}_{\theta_n^{-1}}$ of simple zeroes in $\arg(z) \in (-\pi/3, +\pi/3), n = 71$ and $= 12$.

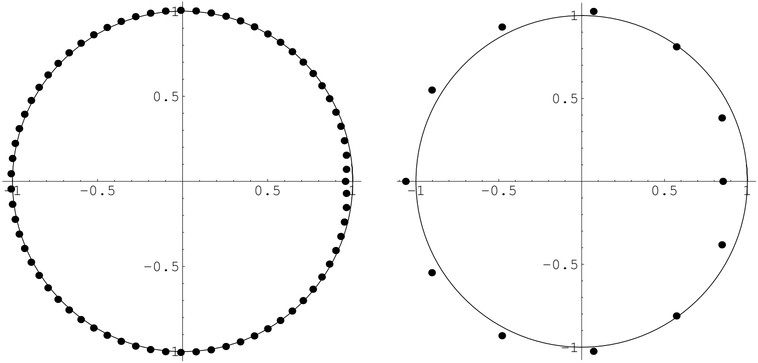

FIGURE: Roots of $G_{71}(z)$, $G_{12}(z)$.

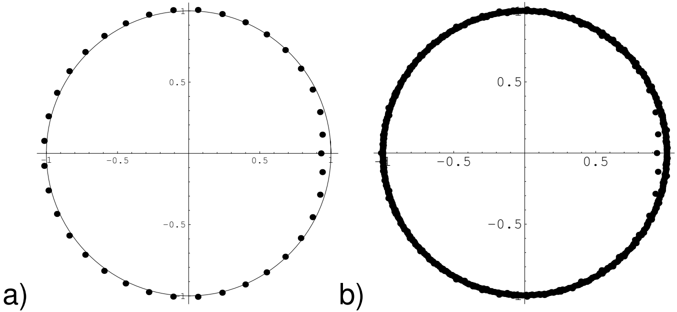

FIGURE: a) The 37 zeroes of $G_{37}(x) = -1 + x + x^{37}$, b) The 649 zeroes of $f(x) = G_{37}(x) + +x^{81} + x^{140} + x^{184} + x^{232} + x^{285} + x^{350} + x^{389} + x^{450} + x^{514} + x^{550} + x^{590} + x^{649} = G_{37}(x) + x^{81} + \ldots + x^{649}$. The lenticulus of roots of f (having 3 simple zeroes) is obtained by a very slight deformation of the restriction of the lenticulus of roots of G_{37} to the angular sector $|\arg z| < \pi/18$, off the unit circle. The other roots (nonlenticular) of f can be found in a narrow annular neighbourhood of $|z| = 1$.

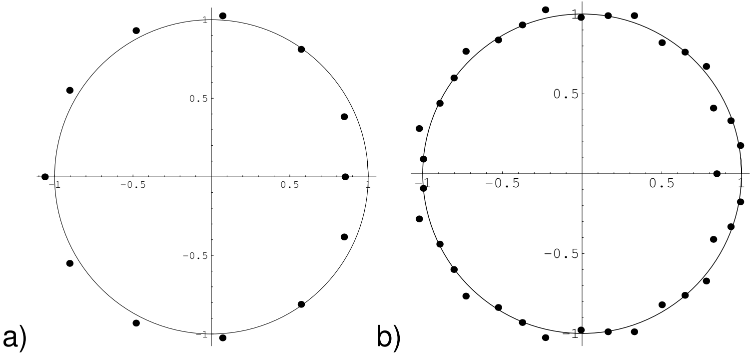

FIGURE: a) The 12 zeroes of G_{12}, b) The 35 simple zeroes of $f(x) = -1 + x + x^{12} + x^{23} + x^{35}$. By definition, only one root is lenticular, the one on the real axis, though the "complete" lenticulus of roots of $-1 + x + x^{12}$, slightly deformed, can be guessed.

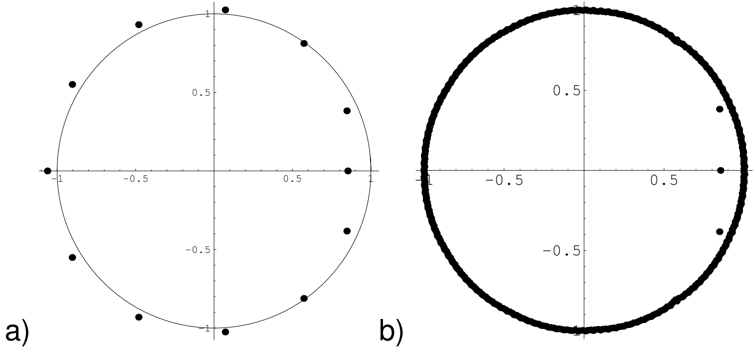

FIGURE: a) The 12 zeroes of G_{12}, b) The 385 zeroes of $f(x) = -1 + x + x^{12} + x^{250} + x^{385}$. The lenticulus of roots of the trinomial $-1 + x + x^{12}$ can be guessed, slightly deformed and almost "complete". It is well separated from the other roots, and off the unit circle. Only one root of f is considered as a lenticular zero, the one on the real axis : 0.8.... The thickness of the annular neighbourhood of $|z| = 1$ which contains the nonlenticular zeroes of f is much smaller than in Figure 4b.

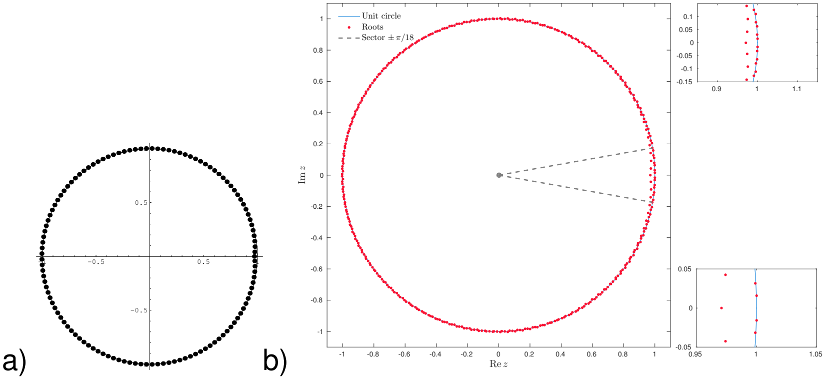

FIGURE: a) Zeroes of G_{121}, b) Zeroes of $f(x) = -1 + x + x^{121} + x^{250} + x^{385}$. On the right the distribution of the roots of f is zoomed twice in the angular sector $-\pi/18 < \arg(z) < \pi/18$. The lenticulus of roots of f has 7 zeroes.

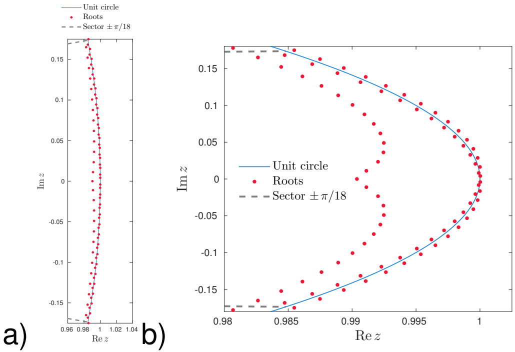

FIGURE: The representation of the 27 zeroes of the lenticulus of $f(x) = -1 + x + x^{481} + x^{985} + x^{1502}$ in the angular sector $-\pi/18 < \arg z < \pi/18$ in two different scalings in x and y (in a) and b)). In this angular sector the other zeroes of f can be found in a thin annular neighbourhood of the unit circle. The real root $1/\beta > 0$ of f is such that β satisfies : $1.00970357\ldots = \theta_{481}^{-1} < \beta = 1.0097168\ldots < \theta_{480}^{-1} = 1.0097202\ldots$.

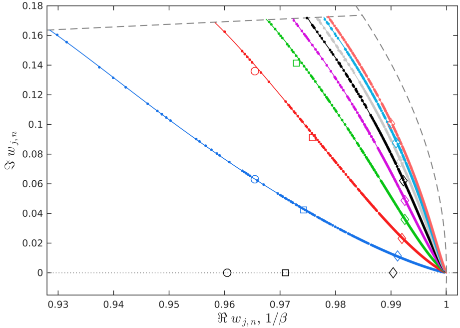

FIGURE: Universal curves stemming from 1 which constitute the lenticular zero locus of all the polynomials of the class $\mathscr{B}$. These curves are continuous, semi-fractal. The first one above the real axis, corresponding to the zero locus of the first lenticular roots, lies in the boundary of Solomyak's fractal [**?**]. The lenticular roots of the previous polynomials f are represented by the respective symbols o, $\square$, $\diamond$. The dashed lines represent the unit circle and the top boundary of the angular sector $|\arg z| < \pi/18$.

Class $\mathscr{B}$ - factorization

context : Schinzel's Theorems "Reducibility of Lacunary Polynomials", I, II, ... over 30 years.

Quadrinomials : Ljunggren (1960), Mills (1985), Finch and Jones (2006).

Theorem

For any $f \in \mathscr{B}_n$, $n \geq 3$, denote by

$$f(x) = A(x)B(x)C(x) = -1 + x + x^n + x^{m_1} + x^{m_2} + \ldots + x^{m_s},$$

where $s \geq 1$, $m_1 - n \geq n-1$, $m_{j+1} - m_j \geq n-1$ for $1 \leq j < s$, the factorization of f where A is the **cyclotomic** *part, B the* **reciprocal noncyclotomic** *part, C the* **nonreciprocal** *part. Then $B = 1$, (i) the nonreciprocal part C is nontrivial,* **irreducible**, *of degree*

$$\deg(C) \geq \lfloor \frac{m_s - 1}{2} \rfloor,$$

and **never vanishes** *on the unit circle, (ii) if $\beta > 1$ denotes the real algebraic number uniquely determined by the sequence $(n, m_1, m_2, \ldots, m_s)$ such that $1/\beta$ is the unique real root of f in (θ_{n-1}, θ_n), $C^*(X)$ is the* **minimal polynomial** *$P_\beta(X)$ of β.*

Pf. : (i) Ljunggren's argument (1960) for the uniqueness of the nonreciprocal factor in C,
(ii) Willson Orchard's reduced form (2005) of Sylvester determinant of the resultant of f and f^* to characterize the reciprocal zeroes of f,
(iii) nonvanishing of f^* on the zeroes of the trinomials $-1 + x + x^n$.

Distinguishability

In the angular sector $-\pi/18 < \arg z < +\pi/18$, any $f \in \mathscr{B}$ admits 2 collections of roots : those (lenticulus) going slowly to 1 in modulus, those (annular neighbourhood) going very fast to 1 in modulus.

Theorem

There exists 2 positive constants $c_1, c_2 < c_1$, such that, for $n \geq 3$, the roots of $f \in \mathscr{B}_n$ lying in $-\pi/18 < \arg z < +\pi/18$ either belong to

$$\{z \mid ||z| - 1| < \frac{c_2}{n}\},$$

or to

$$\{z \mid ||z| - 1| > \frac{c_1}{n}\}.$$

Definition of the **lenticulus** : $\mathscr{L}_\beta := \{z \mid ||z| - 1| > \frac{c_1}{n}\}$. Take roughly $c_1 = 5$ and $c_2 = 1/5$.

Comparison - lenticuli of roots

Trinomials : $|z| < 1 \iff |\arg(z)| < \pi/3.$

all roots in the open unit disk :

$$\#\mathscr{L}_{\theta_n^{-1}} = 1 + 2\lfloor n/6 \rfloor$$

Class $\mathscr{B}$: $f \in \mathscr{B}_n$ $\quad |z| < 1 - \dfrac{c_1}{n}$ and $|\arg(z)| < \pi/18.$

the set of zeroes $Z(f)$ is separated into two parts :

$$Z(f) = \mathscr{L}_\beta \cup (Z(f) \setminus \mathscr{L}_\beta),$$

with

$$\#\mathscr{L}_\beta = C_1 + n\,C_2 \qquad (C_1, C_2 \text{ positive constants})$$

region "aisles of the lenticuli" out of reach : $\pi/18 < |\arg(z)| < \pi/3.$

Contents

Minimal polynomials versus Parry Upper functions

	$\deg(\beta)$	|	$\mathrm{dyg}(\beta)$
before after	P_β	|	$f_\beta(z) = -1/\zeta_\beta(z)$
	$\mathrm{M}(\beta)$	|	?

and $f_\beta(z)$ is a power series of constant term -1 with coefficients in $\{0,1\}$, called **Parry Upper upper function** at β, having the general form, with a lacunarity controlled by the dynamical degree :

$$
\begin{aligned}
f_\beta(z) &= -1 + z + z^{\mathrm{dyg}(\beta)} + z^{m_1} + z^{m_2} + z^{m_3} + \ldots \\
&= G_{\mathrm{dyg}(\beta)} + z^{m_1} + z^{m_2} + z^{m_3} + \ldots
\end{aligned}
$$

with $m_1 \geq 2\,\mathrm{dyg}(\beta) - 1, m_{q+1} - m_q \geq \mathrm{dyg}(\beta) - 1, q \geq 1.$

Identification of the lenticular zeroes as lenticular conjugates : 2 steps

Let $n \geq n_0$. There exists Ω_n an open set strictly $\subset D(0,1)$ whose complement contains all the "annular" roots of any polynomial section of any Parry Upper function $f_\beta(z)$ for any β, $\theta_n^{-1} < \beta < \theta_{n-1}^{-1}$.

$$f_\beta(x) := -1 + x + x^n + x^{m_1} + x^{m_2} + \ldots + x^{m_s} + \ldots$$

where $s \geq 0$, $m_1 - n \geq n-1$, $m_{q+1} - m_q \geq n-1$ for $1 \leq q < s$.

With β **fixed**, convergence towards $f_\beta(z)$ on each compact K in Ω_n occurs for

- the family of polynomial sections of $f_\beta(z)$ [**existence of lenticuli of roots**]
- the family of rewritting polynomials from the minimal polynomial $P_\beta(z)$ to $f_\beta(z)$ [**identification of the lenticuli as lenticuli of conjugates**].

Rewritting polynomials

from "P_β" to "f_β" :

$$P_\beta(X) = 1 + a_1 X + a_2 X^2 + a_3 X^3 + \ldots + a_{d-1} X^{d-1} + X^d, \qquad a_i \in \mathbb{Z}, d \geq 1,$$

(reciprocal $a_i = a_{d-i}$),

left extremity of the rewriting trail (of 1 in base β) :

$$1 = 1 - P_\beta(\frac{1}{\beta}) = \frac{-a_{d-1}}{\beta} + \frac{-a_{d-2}}{\beta^2} + \frac{-a_{d-3}}{\beta^3} + \ldots + \frac{-1}{\beta^d}$$

right extremity of the rewriting trail (of 1 in base β) :

$$1 = \frac{1}{\beta} + \frac{0}{\beta^2} + \frac{0}{\beta^3} + \ldots + \frac{0}{\beta^{n-1}} + \frac{1}{\beta^n} + \frac{0}{\beta^{n+1}} + \ldots = 1 + f_\beta(1/\beta)$$

To the left extremity, add

$$0 = \frac{(a_{d-1}+1)}{\beta} P_\beta(1/\beta).$$

Let

$$A_1(X) := -1 + (a_{d-1}+1)X.$$

We obtain the first intermediate β-representation of 1

$$1 = 1 + A_1(\frac{1}{\beta}) P_\beta(\frac{1}{\beta}) = \frac{1}{\beta} + \frac{a_1(a_{d-1}+1) - a_{d-2}}{\beta^2} + \frac{a_2(a_{d-1}+1) - a_{d-3}}{\beta^3} + \ldots$$

The first term $1/\beta$ is the value of the first polynomial section of $1 + f_\beta(z)$ at $1/\beta$. Then iterate : A_2, A_3, $\ldots$ up till the right extremity.

sequence of rewriting polynomials :

$$(A_s(z)P_\beta(z))_{s\geq 1}$$

Uniform upper bound on the height :
Since all the coefficients of $P_\beta(X)$ and $f_\beta(X)$ are finite, there exists an integer $H \geq H(P_\beta)$ such that the naïve heights of the *rewriting polynomials* $A_s P_\beta$ satisfy

$$H(A_s(X)P_\beta(X)) \leq H \qquad \text{all} \;\; s \geq 1.$$

Therefore

$$\lim_{s\to\infty} A_s(z)P_\beta(z) = f_\beta(z)$$

for the **topology of uniform convergence on every compact** $K \subset \Omega_n$**.**

Indeed, for z belonging to any compact subset $K \subset \Omega_n$,

$$|A_s(z)P_\beta(z) - f_\beta(z)| = \Big|(A_s(z)P_\beta(z) + 1 - \sum_{j=1}^{s} t_i z^j) + (-1 + \sum_{j=1}^{s} t_i z^j - f_\beta(z))\Big|$$

$$\leq (H+1) \sum_{j=s+1}^{\deg(P_\beta)+s} |z|^j + \sum_{j=s+1}^{\infty} |z|^j \leq (H+2)|z|^{s+1} \frac{1}{1-|z|}.$$

2 relations, with uniform convergence on every K in Ω_n,

- existence of lenticuli (each s_N in the class $\mathscr{B}$)

$$\lim_{N\to\infty} s_N(z) = f_\beta(z)$$

-

$$\lim_{s\to\infty} A_s(z)P_\beta(z) = f_\beta(z)$$

Hence, since $f_\beta(z)$ has no zero in Ω_n except the lenticular zeroes, **every zero of $P_\beta(z)$ is a zero of $f_\beta(z)$.**
Conversely, since $1/\beta$ is a simple zero of $P_\beta(z)$ and of $f_\beta(z)$, that lenticuli are limits of the lenticuli of the irreducible factors of the $s_N(z)$, then the **complete lenticulus of $f_\beta(z)$ is a lenticulus of zeroes of $P_\beta(z)$.**

Fracturability of the minimal polynomial, Questions of rationality, Open problems

β is a Parry number iff $\zeta_\beta(z)$ is a rational fraction

iff the unit circle is the natural boundary of $\zeta_\beta(z)$

Define $\Omega_n :=$ open subset of $|z| < 1$ (avoiding a neighbourhood of $|z| = 1$. Then (**fracturability** of the minimal polynomial of β by two infinite integer power series) :

$$P_\beta(z) = (P_\beta(z)/f_\beta(z)) \times f_\beta(z), \qquad z \in \Omega_n.$$

outside : out of reach !

rationality questions : analogues of Dwork's and Kedlaya's approach with *p*-adic methods applied to A. Weil's dynamical zeta function for counting points on varieties defined over finite fields.

1st conjecture of Weil (Dwork),

2nd conjecture of Weil, "Weil II" (Deligne, Kedlaya).


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